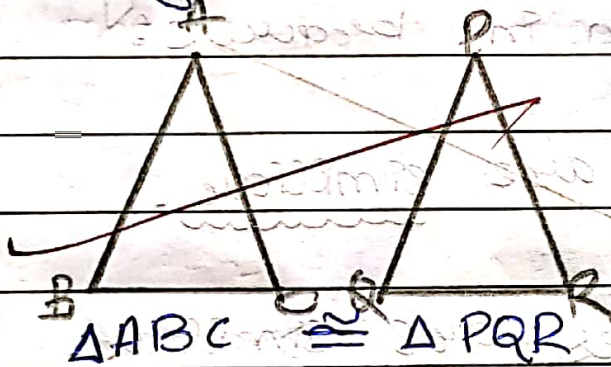


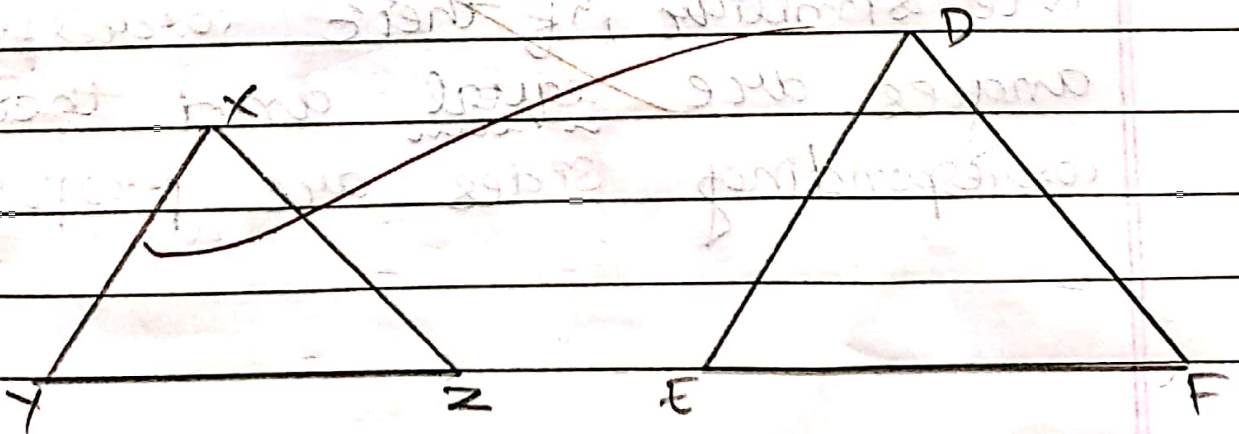
# TRIANGLES

## • Similar Figures & Congruent Figures

Two figures are said to be Congruent if they have same ~~an~~ shape and same size.

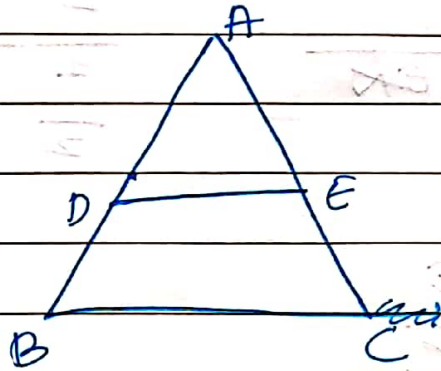


Two figures are said to be similar if they have same shape but different size.



## Basic Proportionality Theorem (BPT)

If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then the other two sides are divided in same ratio.



$$\frac{AD}{DB} = \frac{AE}{EC}$$

Proof :-

Given  $DE \parallel BC$

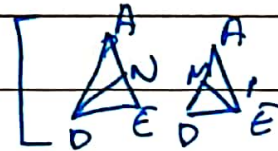
To find  $\frac{AD}{DB} = \frac{AE}{EC}$

Proof :-

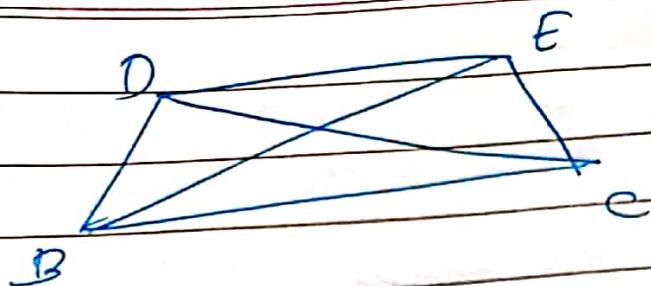
$$A \text{ of } \Delta = \frac{1}{2} \times b \times h$$

$$A \text{ of } \Delta ADE = \frac{1}{2} \times AD \times EN$$

$$= \frac{1}{2} \times AE \times DM$$







$$A \text{ of } \triangle BDE = A \text{ of } \triangle CDE$$

$$\frac{1}{2} \times DB \times EN = \frac{1}{2} \times EC \times DM$$

$$\frac{A(\triangle ADE)}{A(\triangle BDE)} = \frac{A(\triangle ADE)}{A(\triangle CDE)}$$

$$\frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM}$$

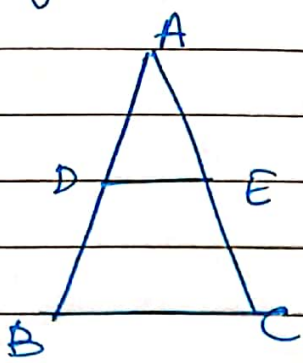
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved

Converse of BPT

~~Proof~~

If a line divides any two sides of a triangle in the same ratio, then the line will be parallel to the third side.



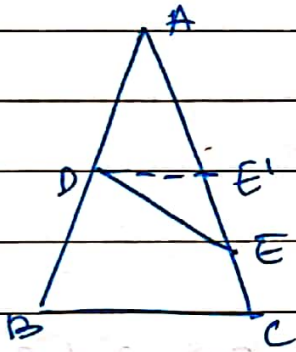
∴

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Then,

$$DE \parallel BC$$

Proof,



Given

$$\frac{AD}{DB} = \frac{AE}{EC}$$

To prove  $DE \parallel BC$

Constr  $\rightarrow$  Draw  $DE' \parallel BC$

Proof



Assume  $DE \parallel BC$

Since  $DE' \parallel BC$

$AD = AE'$  (From BPT)

$$\frac{AD}{DB} = \frac{AE'}{E'C}$$

$$\frac{AE}{EC} = \frac{AE'}{E'C}$$

$$\frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1$$

$$\frac{AE + EC}{EC} = \frac{AE' + E'C}{E'C}$$

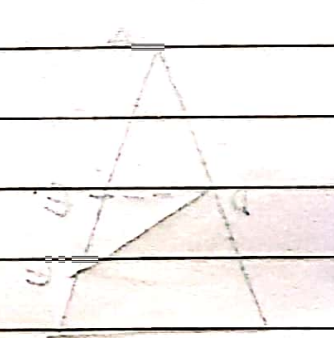
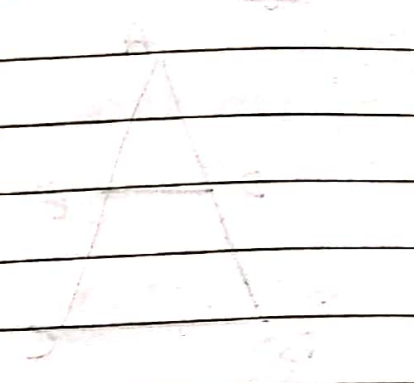
$$\frac{AC}{EC} = \frac{AC}{E'C}$$

$$EC = E'C$$

This implies  $E$  and  $E'$  must coincide

∴  $DE \parallel BC$

Hence proved



## Examples

In  $\triangle ABC$ , if  $DE \parallel BC$ , prove that  $\frac{AB}{AD} = \frac{AC}{AE}$

→ Given,  $DE \parallel BC$

$$\frac{AB}{AD} = \frac{AC}{AE} \quad \text{To prove}$$

Proof,

$$DE \parallel BC$$

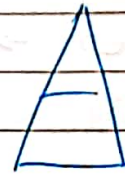
$$\frac{AD}{DB} = \frac{AE}{EC} \quad (\text{From BPT})$$

$$\frac{DB + 1}{AD} = \frac{EC + 1}{AE}$$

$$\frac{DB + AD}{AD} = \frac{EC + AE}{AE}$$

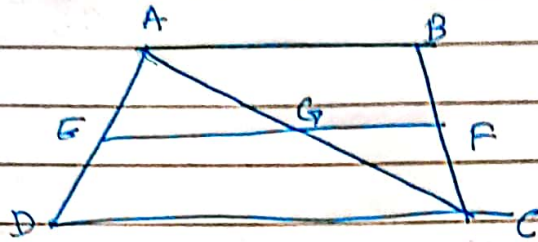
$$\therefore \frac{AB}{AD} = \frac{AC}{AE}$$

Hence proved





② ABCD is a trapezium, with  $AB \parallel DC$  and  $EF \parallel AB$ .  
 Prove that  $\frac{AE}{ED} = \frac{BF}{FC}$



Given  $\rightarrow$   $AB \parallel DC$   
 $EF \parallel AB$

To prove  $\rightarrow$   $\frac{AE}{ED} = \frac{BF}{FC}$

Cons  $\rightarrow$  Join A & C

Proof

Since  $AB \parallel DC$  &  $EF \parallel AB$   
 $EF \parallel DC$

In  $\triangle ADC$ ,  
 $\frac{AE}{ED} = \frac{AG}{GC}$  (From BPT)  $\rightarrow$  ①

In  $\triangle CAB$   
 $\frac{CG}{GA} = \frac{CF}{FB}$  (From BPT)  $\rightarrow$  ②

Compare ① & ②

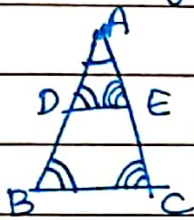
$$\frac{AE}{ED} = \frac{BF}{FC}$$

hence proved

## CRITERIA FOR SIMILARITY

① AAA

Proof,



Given,

$$\angle A = \angle A$$

$$\angle B = \angle D$$

$$\angle C = \angle E$$

To prove

$$\triangle ABC \sim \triangle ADE$$

Proof

In  $\triangle ABC$  &  $\triangle ADE$ ;

$$\therefore \angle D = \angle B \text{ \& \& } \angle E = \angle C$$

$DE \parallel BC$  ( $\angle B$  &  $\angle D$  are corresp)

By BPT

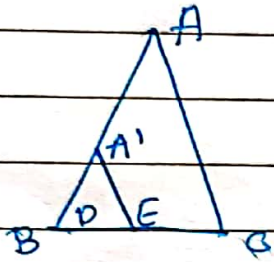
$$\frac{AD}{DB} = \frac{AE}{EC}$$



$$\frac{DB + AD}{AD} = \frac{EC + AE}{AE}$$

$$\frac{DB + AD}{AB} = \frac{EC + AE}{AC}$$

$$\frac{AD}{AB} = \frac{AE}{AC} \quad \text{--- (1)}$$



$A'E \parallel AC$  ( $\angle A' \hat{=} \angle A$  are corresponding angles)

By BPT,

~~$$\frac{AD}{AB} = \frac{DE}{BC}$$~~

$$\frac{A'D}{AB} = \frac{DE}{BC}$$

$$\frac{AD}{AB} = \frac{DE}{BC} \quad (\because BA' = AD \text{ \& } BE = DE) \quad \text{--- (2)}$$

A From (1) & (2)

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$\triangle ABC \sim \triangle ADE$

Hence proved

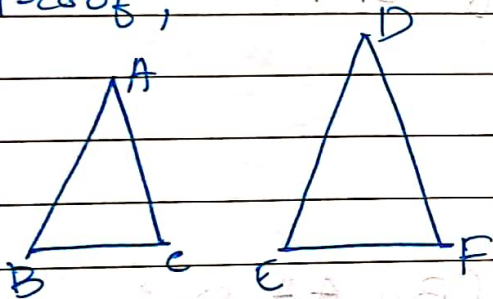
## AA Similarity

If two angles of one triangle are equal to the corresponding two angles of another triangle, then the two triangles are similar.

## SSS Similarity

In two  $\Delta$ 's, if sides of 1  $\Delta$  are proportional to the sides of other  $\Delta$ , then their corresponding angles are equal and hence the two triangles are similar.  $\square$

Proof,



The sides are in same ratio, i.e.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

To prove

$$\angle A = \angle D$$

$$\angle B = \angle E$$

$$\angle C = \angle F$$

Constr

Mark P on DE such DP = AB

Mark Q on DF such DQ = AC

Join PQ



$$\frac{AB}{DE} = \frac{AC}{DF}$$

$$\frac{DP}{DE} = \frac{DQ}{DF}$$

$$\frac{DP + PE}{DE} = \frac{DQ + QF}{DF}$$

$$\frac{DP}{PE} = \frac{DQ}{QF}$$

$PQ \parallel EF$  (Converse of BPT)

$$\angle P = \angle E$$

$$\angle Q = \angle F$$

$\therefore \Delta DPQ \sim \Delta DEF$  (By AA siml)

Hence, sides are proportional

$$\frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF} \quad \text{--- (1)}$$

$AB = DP$  &  $AC = DQ$  (By const)

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} \quad \text{(Given) --- (2)}$$

Step 1 (1) (2)

$$BC = PQ$$

$\therefore \triangle ABC \cong \triangle DPQ$  (By SSS criterion)

$$\therefore \angle A = \angle D$$

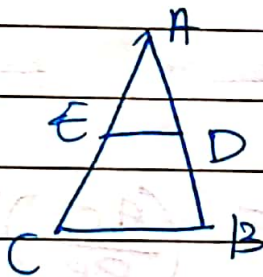
$$\angle B = \angle P = \angle E$$

$$\angle C = \angle Q = \angle F$$

$$\therefore \triangle ABC \sim \triangle DEF$$

SAS criterion

If one angle of a triangle is equal to one angle of other triangle & the sides including these angles are proportional, then the two triangles will be similar.



$$\frac{AD}{AB} = \frac{AE}{AC}$$

Proof,

Given,

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\angle A = \angle A$$

To prove  $\triangle ABC \sim \triangle ADE$



Proof,

$DE \parallel BC$  (converse of BPT)

$$\angle D = \angle B \text{ \& } \angle E = \angle C$$

$\triangle ABC \sim \triangle ADE$  (By AA similarity)

Hence proved

The ratio of the area of two similar triangles is equal to the square of the ratio of their corresponding sides.

Proof,

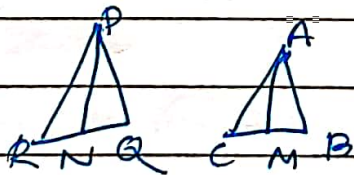


Given:  $\triangle ABC \sim \triangle PQR$

To prove

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

Const<sup>n</sup>  $\rightarrow$   $AM \perp BC$  &  $PN \perp QR$



Proof,

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$$

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{BC \times AM}{QR \times PN}$$

$$\frac{1}{2} \times QR \times PN$$

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AM}{QR \times PN} \quad \text{--- (1)}$$

In  $\Delta ABM$  &  $\Delta PQN$

$$\begin{aligned} \angle B &= \angle Q & (\Delta ABC \sim \Delta PQR) \\ \angle M &= \angle N & (\text{Right angle}) \end{aligned}$$

$\therefore \Delta ABM \sim \Delta PQN$  (By AA sim)

By

$$\frac{AM}{PN} = \frac{AB}{PQ} \quad \text{--- (2)}$$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad \text{--- (3)}$$

Replace (2) & (3) in (1)

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB \times AB}{PQ \times PQ} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

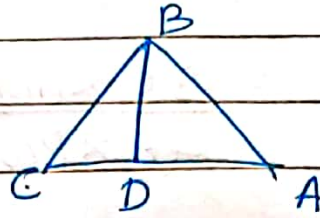
Hence proved

~~Dist~~

In a right angle triangle, if a perpendicular is drawn from the vertex of the right angle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and to each other.



Given  $\angle B = 90^\circ$   
 $BD \perp AC$



To prove,

$$\triangle ADB \sim \triangle ABC$$

$$\triangle BDC \sim \triangle ABC$$

$$\triangle ADB \sim \triangle BDC$$

Proof

In  $\triangle ADB$  &  $\triangle ABC$ ,

$$\angle A = \angle A \text{ (Common)}$$

$$\angle ADB = \angle ABC \text{ (Right angle)}$$

$$\therefore \triangle ADB \sim \triangle ABC \text{ (AA sim)}?$$

In  $\triangle BDC$  &  $\triangle ABC$ ,

$$\angle C = \angle C \text{ (Common)}$$

$$\angle BDC = \angle ABC \text{ (Right angle)}$$

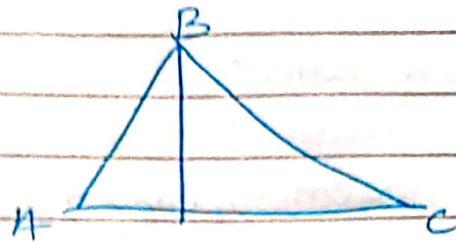
$$\therefore \triangle BDC \sim \triangle ABC \text{ (AA sim)}$$

$$\Rightarrow \triangle ADB \sim \triangle BDC \text{ (Inner } \triangle \text{ are similar)}$$

# Pythagoras Theorem

Given.

$$\angle B = 90^\circ$$



$$\text{T.P. } AB^2 + BC^2 = AC^2$$

Cons Draw  $BD \perp AC$

Proof,

$$\triangle ADB \sim \triangle ABC$$

$$\frac{AD}{AB} = \frac{AB}{AC}$$

$$AD \times AC = AB^2 \quad \text{--- (1)}$$

$$\triangle BDC \sim \triangle ABC$$

$$\frac{CD}{BC} = \frac{BC}{AC} \quad \text{--- (2)}$$

$$CD \times AC = BC^2 \quad \text{--- (3)}$$

From (1) + (2)

Add,

$$AB^2 + BC^2 = AC (AD + CD)$$

$$AB^2 + BC^2 = AC^2$$

Hence proved





In  $\triangle ABC$ , if  $\angle B = 90^\circ$  &  $BD \perp AC$ , then  $BD^2 = AD \times DC$

Given  $BD \perp AC$

~~To prove~~  $\angle ABC = 90^\circ$

To prove  $BD^2 = AD \times DC$

Proof

$\triangle ADB \sim \triangle ABC$

$\triangle BDC \sim \triangle ABC$

Hence,  $\triangle ADB \sim \triangle BDC$

$$\Rightarrow \frac{AD}{BD} = \frac{BD}{DC}$$

$$AD \times DC = BD^2$$

Hence proved

# CRITERIA FOR SIMILARITY OF TRIANGLES

If two triangles are similar then, their corresponding sides are in proportion and corresponding angles are equal.

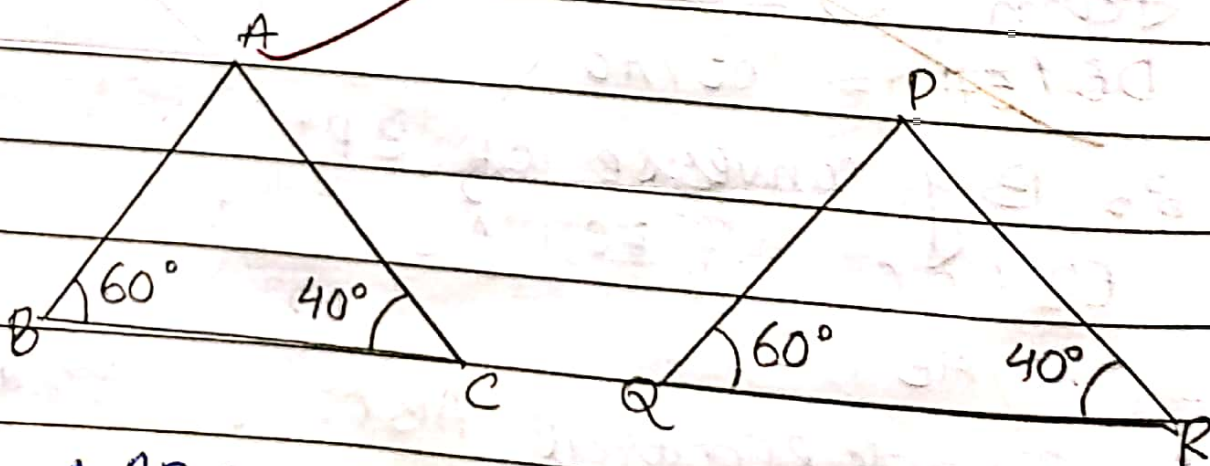
There are three criteria for similarity of triangles :-

① AA criteria

② SSS criteria

③ SAS criteria

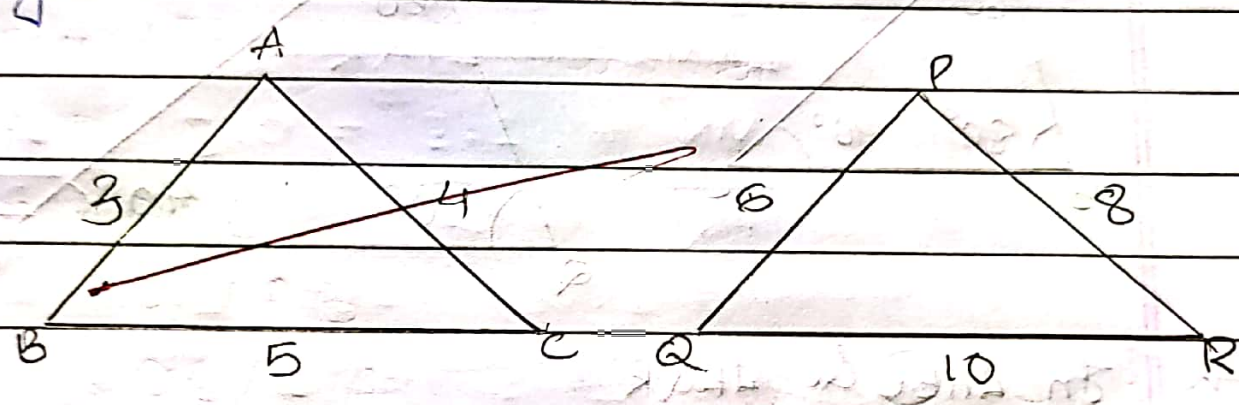
**THEOREM 6.3** :- If in two triangles, corresponding angles are equal, then their corresponding sides are in same ratio (or proportion) and hence the two triangles are similar.



$\triangle ABC \sim \triangle PQR$

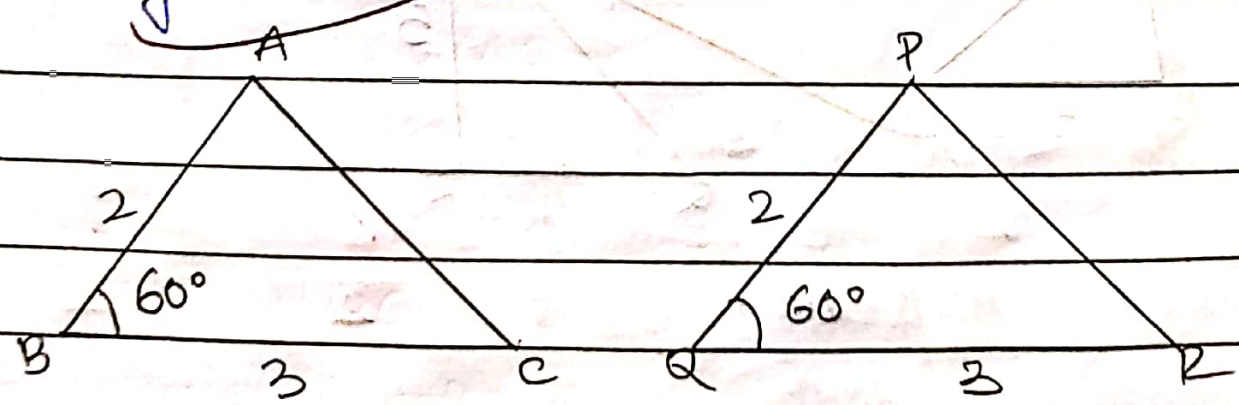


**THEOREM 6.4 :-** If in two triangles, sides of one triangle are proportional to (i.e. in same ratio) the sides of other triangle, then their corresponding angles are equal and hence the two triangles are similar.



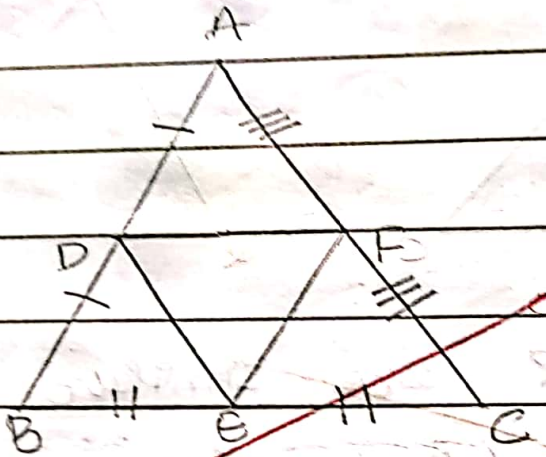
$\triangle ABC \sim \triangle PQR$

**THEOREM 6.5 :-** If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.



Thus,  $\triangle ABC \cong \triangle PQR$   
By SSS criterion

D, E and F are respectively the mid-points of sides AB, BC and CA of  $\triangle ABC$ . Find the ratio of areas of  $\triangle DEF$  and  $\triangle ABC$ .



D, E and F are mid points of AB, BC and AC of  $\triangle ABC$ .



By Mid-point theorem,

$$DF = \frac{1}{2} BC, \quad DE = \frac{1}{2} AC, \quad EF = \frac{1}{2} AB$$

$$\frac{DF}{BC} = \frac{1}{2}, \quad \frac{DE}{AC} = \frac{1}{2}, \quad \frac{EF}{AB} = \frac{1}{2}$$

$$\therefore \frac{DF}{BC} = \frac{DE}{AC} = \frac{EF}{AB} \quad \text{--- (1)}$$

In  $\triangle DFE$  &  $\triangle ABC$ ,

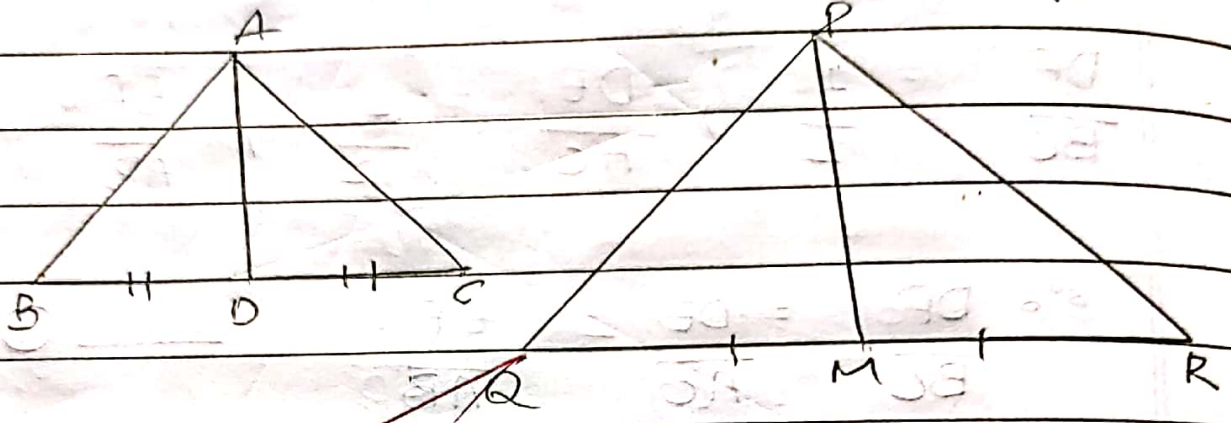
$$\frac{DF}{BC} = \frac{DE}{AC} = \frac{EF}{AB} \quad \text{--- from (1)}$$

$$\therefore \triangle DFE \sim \triangle ABC$$

$$\therefore \frac{\text{ar}(\triangle DFE)}{\text{ar}(\triangle ABC)} = \frac{DF^2}{BC^2} = \frac{(1)^2}{(2)^2} = \frac{1}{4}$$

$$\therefore \text{ar}(\triangle DFE) \text{ is } \text{ar}(\triangle ABC) \\ 1 \text{ is } 4$$

Q. Prove that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding medians.



GIVEN :-  $\triangle ABC \sim \triangle PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad \text{--- (1)}$$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$$

~~$$\frac{AB}{PQ} = \frac{BD}{QM}$$~~

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad \text{--- (2)}$$



In  $\triangle ABD$  &  $\triangle PQM$ ,

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad \text{--- from (2)}$$

$$\angle B = \angle Q \quad \text{--- from (1)}$$

$\therefore \triangle ABD \sim \triangle PQM$  By SAS criteria

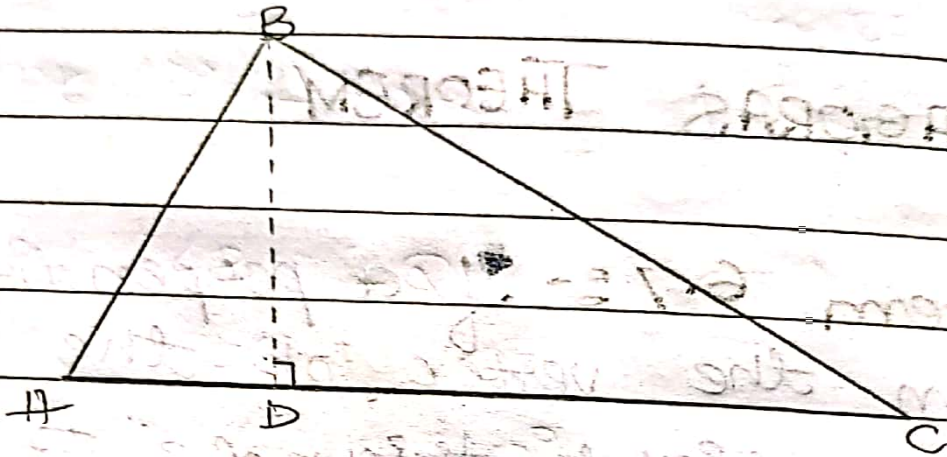
$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \quad \text{--- (3)}$$

$$\begin{aligned} \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} &= \frac{AB^2}{PQ^2} = \left(\frac{AB}{PQ}\right)^2 \\ &= \left(\frac{AD}{PM}\right)^2 \quad \text{--- from (3)} \\ &= \frac{AD^2}{PM^2} \end{aligned}$$

Hence proved.

## THEOREM 6.8 :-

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Given,  $\angle B = 90^\circ$

To prove  $AC^2 = AB^2 + BC^2$

Draw  $BD \perp AC$

$\triangle ADB \sim \triangle ABC$  ( $BD \perp AC$ )

So,

$$\frac{AD}{AB} = \frac{AB}{AC}$$

$$AD \times AC = AB^2 \quad \text{--- (1)}$$

$\triangle BDC \sim \triangle ABC$  ( $BD \perp AC$ )

$$\frac{CD}{BC} = \frac{BC}{AC}$$



$$CD \times AC = BC^2 \quad \text{--- (2)}$$

Adding (1) & (2)

$$AD \times AC + CD \times AC = AB^2 + BC^2$$

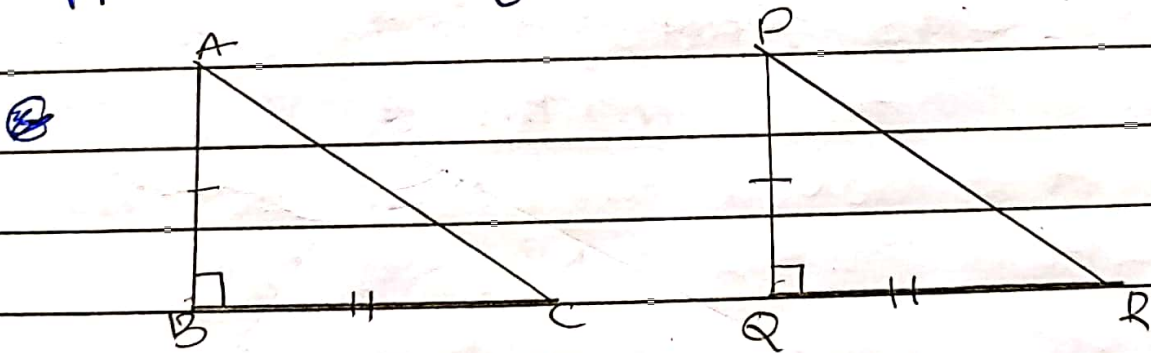
$$AC (AD + CD) = AB^2 + BC^2$$

$$AC \times AC = AB^2 + BC^2$$

$$AC^2 = AB^2 + BC^2$$

THEOREM 6.9 :-

In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.



Given :-  $AC^2 = AB^2 + BC^2$

To prove :-  $\angle B = 90^\circ$

By Pythagoras theorem,

In  $\triangle PQR$

$$PR^2 = PQ^2 + QR^2$$

$$RP^2 = AB^2 + BC^2$$

$$AC^2 = AB^2 + BC^2 \quad (\text{Given}) \quad \text{--- (2)}$$

So,  $AC = PR$  ~~---~~ (2) ~~प्रमाणित~~

In  $\triangle ABC$  &  $\triangle PQR$  ~~+  $\angle A = \angle P$~~

$$AB = PQ \quad (\text{Construction})$$

$$BC = RQ \quad (\text{Construction})$$

$$AC = PR \quad (\text{Proved above (2)})$$

$\therefore \triangle ABC \cong \triangle PQR$  by SSS criteria

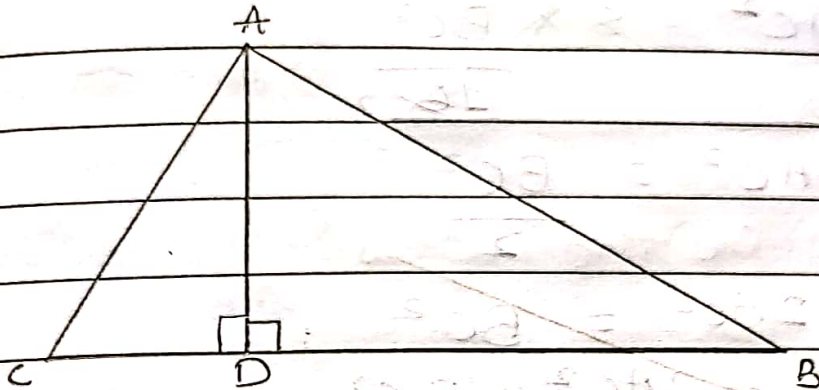
$$\angle B = \angle Q \quad (\text{CPCT})$$

$$\angle Q = 90^\circ$$

$$\angle B = 90^\circ$$



14) The perpendicular from A on side BC of a  $\triangle ABC$  intersects BC at D such that  $DB = 3CD$ . Prove that  $2AB^2 = 2AC^2 + BC^2$ .



→  $DB = 3CD$

$BC = CD + DB$

$BC = CD + 3CD$  (given)

$BC = 4CD$

In  $\triangle ADB$  &  $\triangle ADC$ ,

By Pythagoras theorem,

$AB^2 = AD^2 + DB^2$  — (1)

$AC^2 = AD^2 + CD^2$  — (2)

Subtract

$AB^2 - AC^2 = DB^2 - CD^2$

$AB^2 - AC^2 = (3CD)^2 - CD^2$  (given)

$$AB^2 - AC^2 = 9CB^2 - CD^2$$

$$AB^2 - AC^2 = 8CD^2$$

$$AB^2 - AC^2 = 8 \times \left(\frac{BC}{4}\right)^2$$

$$AB^2 - AC^2 = \frac{8 \times BC^2}{16}$$

$$AB^2 - AC^2 = \frac{BC^2}{2}$$

$$2AB^2 - 2AC^2 = BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$